SAMPLING SIGNATURE INDUCED CAUSALITY CHAIN IN A SET OF TIME SERIES

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MOTIVATIONS

Discovering the underlying topological structure describing the statistical dependencies inside a set of time series is a problem that has recently emerged as an exciting topic in the recent years. In this poster contribution, we address the problem of sampling topological complexes that leverage the expressive power of Signatures, a new feature map invented by Terry Lyons [1, 3] and his collaborators in order to devise a relevant joint distribution. More precisely, following recent work of Giusti and Lee, we use the computed Signatures to understand the causality chains in the time series, that we aggregate into a random complex using sampling proportionally to the causality measure, combined with a noninformative prior enforcing the absence of cycles in the resulting causal directed graph. Signatures of order 2 build graphs and signatures of higher order combine into potentially higher order complexes. Sampling is performed using a simple Metropolis-Hastings algorithm.

SIGNATURE OF ORDER 2

In order to understand causality between two time series X^i and X^j, and following Giusti and Lee [2], we compute $\frac{d}{dt}(S^{ij} - S^{ji}) = X^i dX^j - X^j dX^i$ on a window of time. This quantity represents the signed area of the projected path in $\mathbb{R}^2$ with respect to X^i and X^j coordinates. If X^i and X^j exhibit a lead-lag relationship, the signed area undergoes a significant increase. Consequently, its magnitude can serve as an indicator of causality : a positive value implies that X^i lead X^j, while a negative value signifies the opposite relationship (lag).

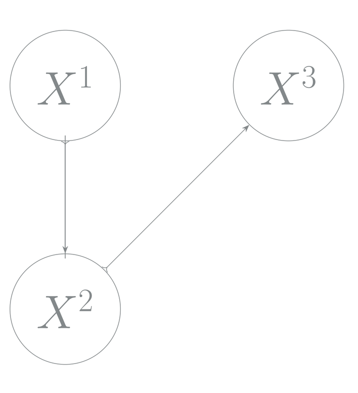
CAUSALITY GRAPH

The primary objective of our algorithm is to construct a causal diagram. To accurately express causality, the graph must be a Bayesian network (a directed acyclic graph). As elucidated in Part 1, the signed area naturally gives direction to the edges. However, assessing causality through signatures is not a definitive test. This serves as a preprocessing step to subsequently evaluate causality using a test (such as Granger causality).

Example : Consider 3 time series (X^1, X^2, X^3).

- Preprocess (X^1, X^2, X^3) to reduce mean to $(0, 0, 0)$
- Compute $\{A(1,2), A(1,3), A(2,3)\}$
- We obtain positive and substantial values for $A(1,2)$ and $A(2,3)$, raising suspicion of causality from X^1 to X^2 and from X^2 to X^3. We can create these two edges.
- The coefficient $A(1,3)$ is negative and lower in magnitude. Establishing causality from X^3 to X^1 would introduce cyclic dependencies into the graph. Given the lower amplitude of $A(1,3)$, we consider this relationship negligible in comparison to the others.
  Thus, the resulting graph is as follows :

Causality Diagram



Remark 1 : In this short example, we consider $A(1,3)$ even if it is negative. In our algorithm, considering that we compute $A(1,3)$ and $A(3,1)$ and that $A(1,3) = -A(3,1)$, we keep the positive one and fix the negative to 0.

Remark 2 : The data preprocessing is inspired from Giusti and Lee.

USING METROPOLIS HASTINGS TO GENERATE THE CAUSAL DIAGRAMS DISTRIBUTION

To sample from the causal diagrams distribution, we use the following Metropolis Hastings algorithm :

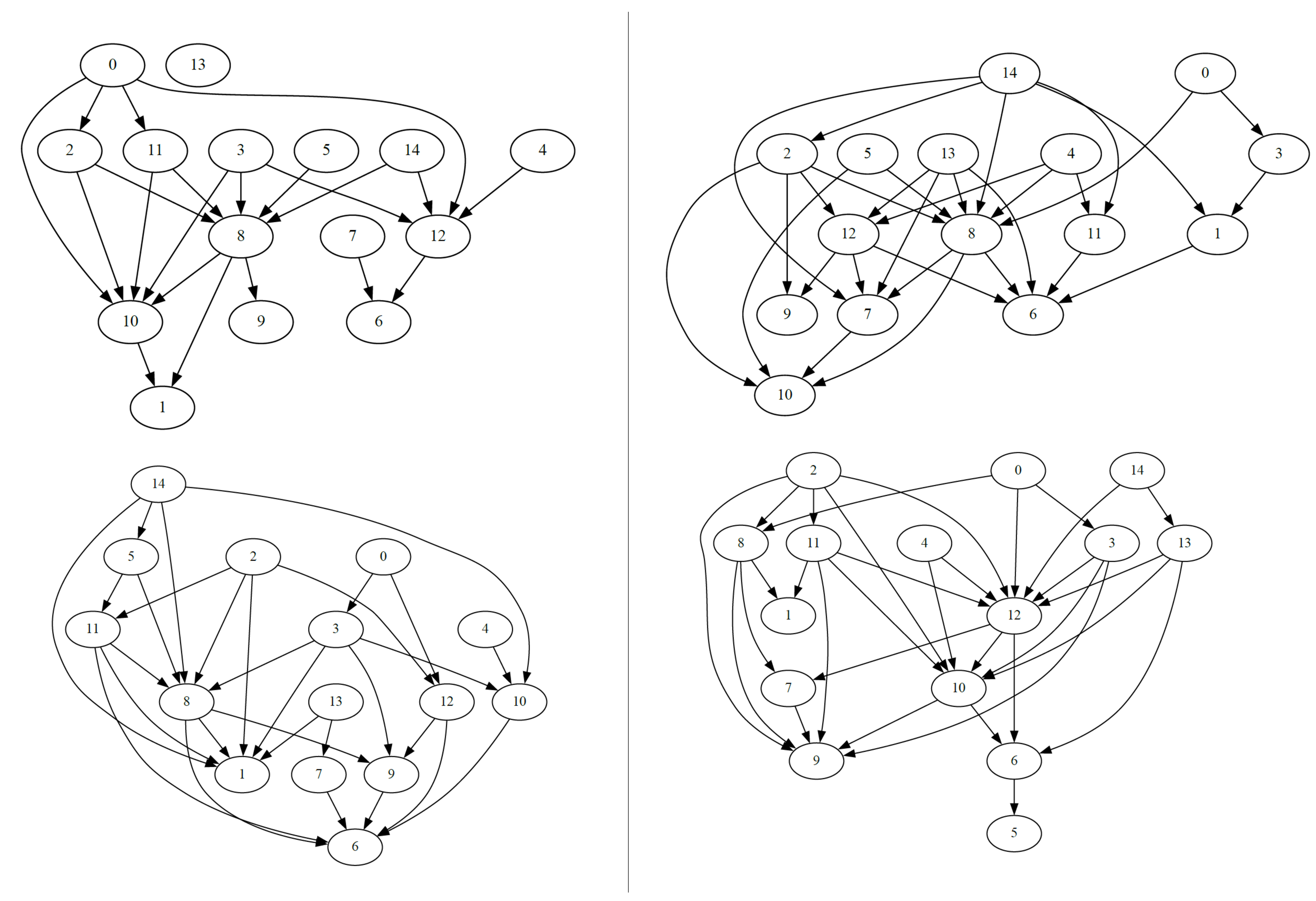
Algorithm 1 Metropolis-Hastings for Causality Diagrams

Set $X = (X^1, X^2, ..., X^n)$.
Compute $P = (S^{ij}(X) - S^{ji}(X))_{ij}$.
Set all $P_{ij} < 0$ to 0.
Normalize P in $[0, 1]$
for k in $\{1, ..., K\}$ **do**
  while $i = j$ **do**
    Sample $i, j \sim \mathcal{U}(\{1, ..., n\})$
  end while
  if $P_{ij} < P_{ji}$ **then**
    $p = 0$
  else
    $p = P_{ij}$
  end if
  Sample $u \sim \mathcal{U}([0, 1])$
  if u<p **then**
    if The diagram with $X^i \to X^j$ is acceptable **then**
      Create the causality arrow $X^i \to X^j$
    else
      Do Nothing
    end if
  else
    Do Nothing
  end if
end for

Remark : In this algorithm, we test our diagram to see if it is acceptable. In this context, acceptable means "respect the causality rules". We test with the package `networkx` from Python if the graph is acyclic.

EXPERIMENTS

We tested our algorithm on the initial time steps of the "fMRI" dataset. The initial samples appear coherent, in the sense that the starting and ending points of the causality chain are relatively stable.



FURTHER WORKS

This algorithm sample from the causality diagram distribution on a fixed window. A further work would be to sampling from the causality distribution on a moving window.

Références

[1] Ilya CHEVYREV et al. "A primer on the signature method in machine learning". In : *arXiv preprint arXiv :1603.03788* (2016).
[2] Chad GIUSTI et al. "Iterated integrals and population time series analysis". In : *Topological Data Analysis : The Abel Symposium 2018*. Springer. 2020, p. 219-246.
[3] Terry J LYONS. "Differential equations driven by rough signals". In : *Revista Matemática Iberoamericana* 14.2 (1998), p. 215-310.

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